PHASE NOISE SHAPING VIA FORCED NONLINEARITY IN PIEZOELECTRICALLY ACTUATED SILICON MICROMECHANICAL OSCILLATORS

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ABSTRACT

This paper shows improved phase-noise performance of MEMS oscillators when the sustaining amplifier operates a lateral bulk acoustic wave AlN-on-Si resonator in the nonlinear regime. An empirical exponential-series-based model that closely describes the phase noise in nonlinearity is presented, reflecting the increased resonator filter order and the reduced amplifier flicker-noise contribution. An oscillator using a 23MHz in-plane shear mode resonator with a quality factor of 4,000 exhibits a phase noise of -130dBc/Hz at 1KHz offset-frequency, corresponding to an improvement of about 20dB with respect to linear operation.

INTRODUCTION

High-performance, low phase-noise oscillators form a fundamental component of modern communication systems, where miniaturization has become a critical design requirement. Research has focused on building microresonators that can be integrated into compact packages and effectively replace large off-chip crystals [1].

Previous work has shown that capacitively-transduced devices can meet tight phase noise (PN) specifications when operated in their linear region [2]. However, their use of large DC polarization voltages introduces system design challenges. Alternatively, laterally-excited thin-film piezoelectric-on-silicon resonators can be used at higher operating frequencies with lower motional resistances. They do not require a polarization voltage, which simplifies the interface electronics and reduces power and floorplan area.

It has been shown theoretically that nonlinear operation of a resonator can suppress the flicker noise of the interface circuitry and improve the oscillator phase noise [3]. Although piezoelectric devices do not typically show transduction-induced nonlinearity as observed in capacitive devices, they can be forced into the nonlinear elastic regime when sufficient power is applied to the electrodes.

This work demonstrates that a sustaining amplifier with high transimpedance gain is capable of exploiting nonlinear effects in a bulk-mode piezoelectric resonator to improve close-to-carrier phase noise. Better PN performance is attributed to an increased order of the resonator filtering action due to reduced bandwidth and reduced sensitivity to phase variations. A new empirical model is proposed to better capture these effects due to the inability of the linear Leeson's formula in accurately predicting the measured results.

PIEZOELECTRIC MEMS RESONATOR CHARACTERIZATION

A composite lateral aluminum-nitride-on-silicon (AlN-on-Si) resonator is employed in this work. The device is fabricated from a 10μm thick silicon-on-insulator (SOI) substrate. The resonator stack composition is shown in Figure 1(a), and it can be described as a two-finger electrode design on top of a plate with lateral dimensions of 156μm ×250μm. The layer thicknesses are 9/2/0.1/1/0.1μm of $Si/SiO_2/Mo/AlN/Mo$, respectively. Figure 1(b) shows an SEM image of the final device, and the fabrication process can be found in [4].

The 3D structure of the resonator was simulated with COMSOL finite element software. The simulated S_{21} response agrees closely with device measurements taken with an Agilent E5071C vector network analyzer (VNA). The resonance peaks of interest are identified as an in-plane shear (IPS) mode at 23MHz and a longitudinal extensional (LE) resonance mode at 26MHz. The IPS and the LE modes are electrically 180° out of phase with respect to one another. These mode shapes are presented in Figure 2.

The support tether locations match the zero displacement nodes of both mode shapes minimizing support loss. The displacement patterns lie mainly in the device layer plane, with smaller maximum out-of-plane to in-plane displacement ratio for the IPS mode than the LE mode (0.135 vs. 0.214, respectively). Normalized volume dilation $(\Delta V/|u|_{max})$ of the IPS mode is $4.3\times10^{-6}\mu m^3/\mu m$ compared with 3.4×10^{-3} for the LE mode. Such isovolumetricity indicates that the IPS resonance mode exhibits nearly pure planar shear strain.

As the source power is increased, the device enters the nonlinear operation regime due to geometric and high-order stiffness effects. The nonlinearity causes the peak of the frequency response to bend and shift toward lower (spring-softening) or higher (spring-hardening) frequencies. Several factors influence the bending direction of the peak. The selection of materials, and thickness and configuration of the top electrode determine whether the frequency response exhibits hardening or softening [5].

For the IPS mode in the orientation depicted in Figure 1(b), the acoustic velocity (and consequently frequency) is mostly determined by $0.5(C_{11}-C_{12})$, in contrast with the LE mode whose wave velocity is controlled by the combination $0.5(C_{11}+C_{12}) + C_{44}$ [6]. C_{IJ} designates a silicon stiffness tensor component (the dominant material in the resonator stack). Since the relevant third order elastic stiffness

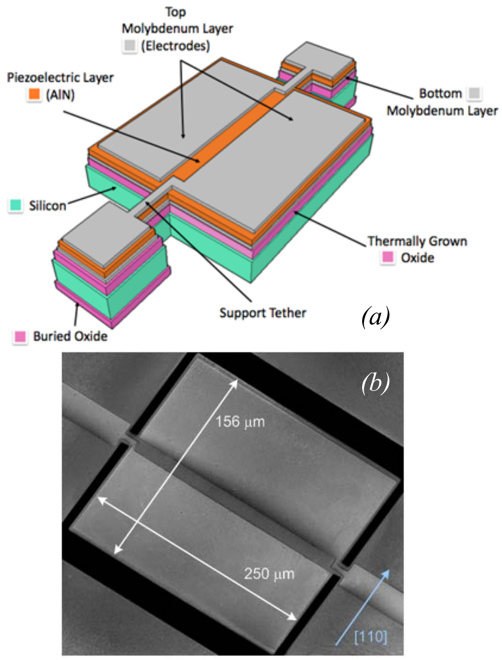

Figure 1: AlN-on-Si MEMS resonator: (a) Schematic illustrating the material layers of the composite stack and (b) SEM view.

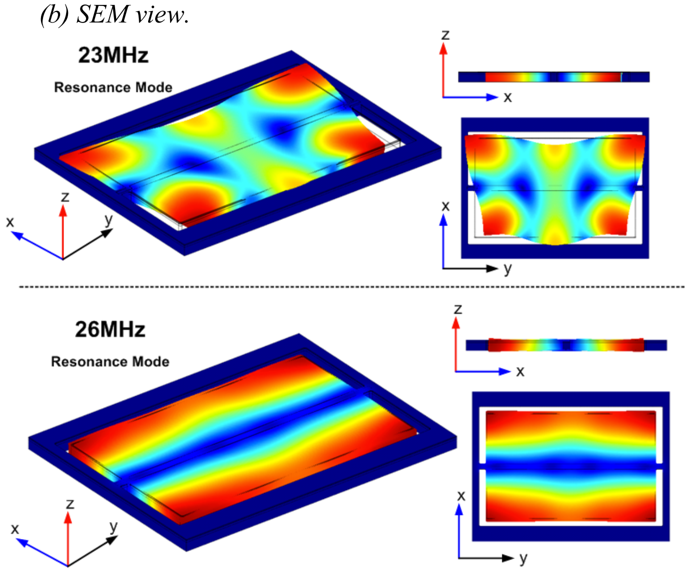

Figure 2: FEM simulations of the IPS (top, 23MHz) and LE (bottom, 26MHz) modes in the two-finger electrode design.

components of silicon (C_{111}, C_{112}, and C_{123}) are negative, it primarily spring-softening is induced by modifications to C_{11} and C_{12} at large strain levels in both modes of the device under consideration [7]. Figure 3 shows that the onset of nonlinearity in the 23MHz IPS mode occurs close to 5dBm and the device can withstand power levels of at least +25dBm. A similar trend is observed for the 26MHz LE mode, but the IPS mode is more susceptible to nonlinearity because the controlling combination of stiffness components is approximately one fourth that of the LE mode (51GPa vs. 195 GPa). The LE mode fractures at about +20dBm, failing at the corners of the support tethers. Although many factors, such as surface roughness and fracture plane orientation [8]

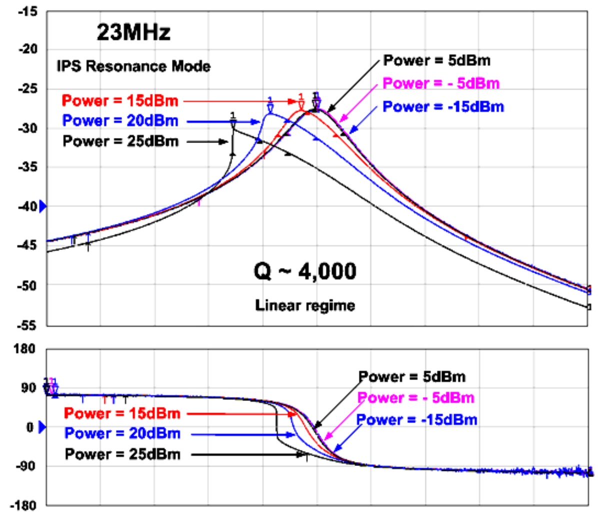

Figure 3: Magnitude and phase frequency plots of the resonator 23MHz-IPS mode for increased power levels.

can affect the fracture strength of silicon, FEM simulations show that, at these corners, the LE mode has strain energy concentrations 26 times larger than the IPS mode at equal input excitation. Therefore, the IPS mode can sustain nonlinear operation at higher power levels.

TRANSIMPEDANCE AMPLIFIER DESIGN

An inverter-based transimpedance amplifier (TIA) with active feedback resistor is interfaced with the MEMS resonator to analyze the nonlinearity effects on the oscillator PN (Fig. 4).

The transimpedance gain is close to 100dBΩ, sufficient to induce nonlinear effects in the piezoelectric resonator. While gain of the input stage is increased by reducing V_{CTRL}, the associated dominant pole is simultaneously moved to lower frequencies. This behavior is used to fine-tune the TIA phase-shift providing a method to move along the resonance curve (Fig. 3).

Figure 5 presents the micrograph of the IC fabricated in a 0.5μm 2P3M CMOS process interfaced with the resonator through minimum-length aluminum bondwires.

NONLINEAR RESONATOR PHASE-NOISE

Depending on the strength of the nonlinearity, the resonator phase-frequency relationship can change from

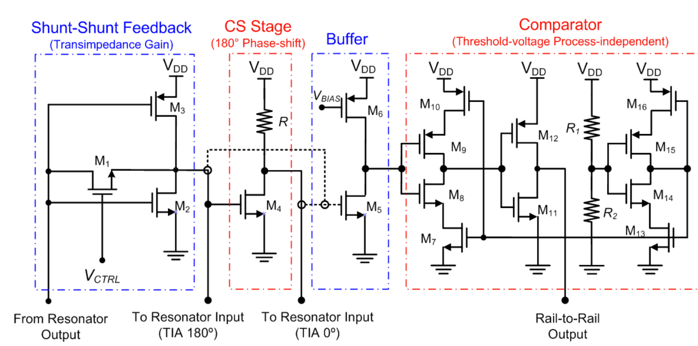

Figure 4: Transimpedance Amplifier Schematic. The TIA provides ports to interface both resonant modes.

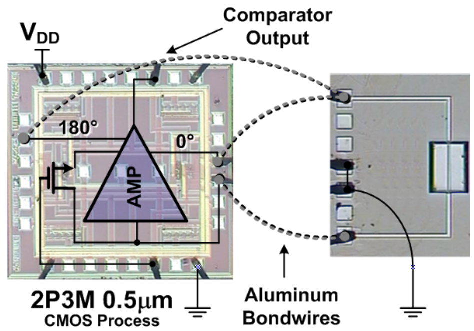

Figure 5: Oscillator comprising the TIA interfaced with the piezoelectric MEMS resonator die using wirebonds.

single-valued to multi-valued. When the slope becomes infinite, the oscillator is insensitive to frequency instability caused by phase jitter. The critical phase is determined as the phase of the resonance mode in linear operation plus 30° for spring-softening behavior (Fig. 6) [3, 9].

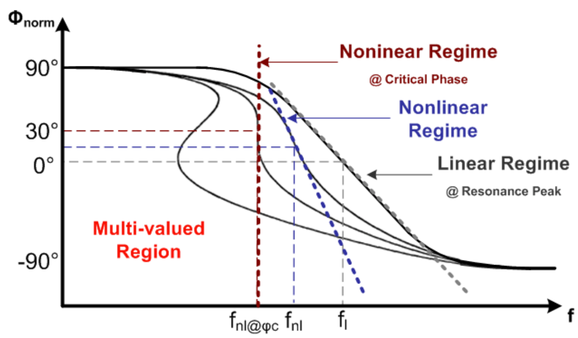

Figure 6: Operating slopes of the phase-frequency relationship as the device enters the nonlinear regime.

To analyze the effect of nonlinearity in the PN performance, oscillators are configured with the 23MHz IPS and 26MHz LE modes using the TIA 180° and 0° ports, respectively. The PN is measured with an Agilent E5500 PN test set for different values of V_{CTRL}. The measurements are taken at the comparator output to limit amplitude variations and keep the signal power constant during the process.

First, the TIA gain is set to operate the resonator linearly, which allows the estimation of Q_{loaded} and flicker noise corner frequency using Leeson's model. Second, V_{CTRL} is reduced gradually to increase the gain and operate the resonator into nonlinear regime. During this test, the phase-shift provided by the TIA simultaneously moves toward the critical value minimizing the phase-sensitivity of the system. The PN results are summarized in Figure 7.

A -30dB/dec slope line is included to reflect the highest possible value predicted by Leeson's. As the resonator is driven into the nonlinear regime, the PN plots *bend* under the linear reference, showing that the oscillator is experiencing a higher effective operating quality factor than Q_{loaded}, as inferred from Figure 6.

To eliminate any contribution from the electronics as the cause of the steeper PN slopes, a second resonator with

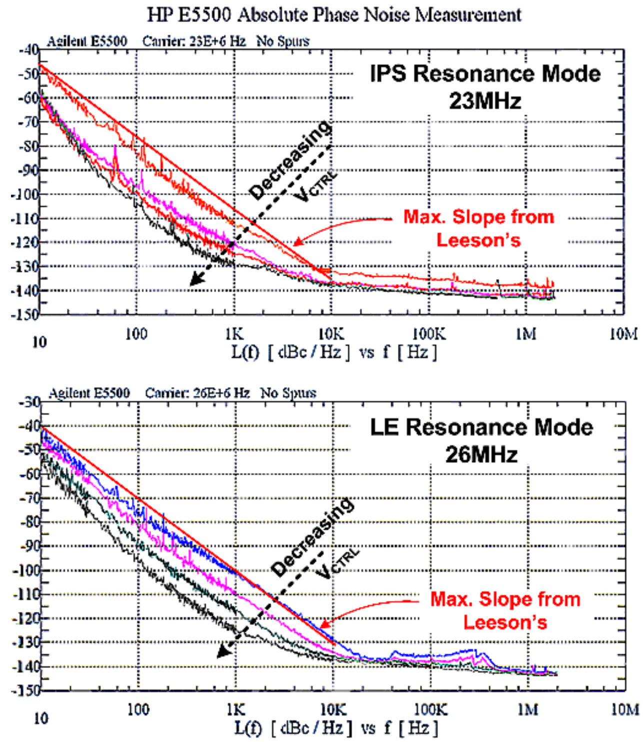

Figure 7: Piezoelectric MEMS-based oscillator PN performance operating in air at room temperature.

similar stack composition and dimensions, but with a reduced-area electrode was interfaced with the same TIA. Device characterization over increased power levels shows a consistently linear operation, with PN results closely fitted with Leeson's model. This test confirms that mechanical nonlinearity causes the improved PN for the first resonator.

NONLINEAR PN MODEL FORMULATION

The Leeson's model predicts three sections on the PN plot for high Q resonators: two segments with slopes equal to -30dB/dec and -10dB/dec, and the noise-floor. Thus, when the PN results show steeper close-to-carrier slopes (as in Fig. 7), an alternative model with higher order terms is required.

The general form of a PN model in terms of power spectral densities [12] can be expressed as

$$M(\omega) = \left[(H(j\omega)-1)\cdot(H^*(j\omega)-1)\right]^{-1} \quad (1)$$

$$S_{\phi_i}(\omega) = S_{\phi_o}(\omega)\cdot M(\omega). \quad (2)$$

where $S_{\Phi i}(\omega)$ and $S_{\Phi o}(\omega)$ are the one-side densities of the phase uncertainties at the input and output of the TIA, and $H(j\omega)$ is the resonator equivalent low-pass transfer function.

Since the nonlinearity is attributed to the MEMS device, $S_{\Phi o}(\omega)$ remains dependent on the flicker and thermal noise of the amplifier by definition. Hence, $H(j\omega)$ must include the higher order terms to describe the results of Figure 7. A sharper equivalent resonator response will reflect the phase-to-frequency slope trend as it is driven deeper into nonlinear regime (Fig. 6).

$$L(f)=\frac{FkTB}{2P}\cdot\left[1+\frac{f_{c_{eff}}}{f}\right]\cdot\left[1+\frac{f_0}{f}\cdot\frac{1}{4Q_{eff}}+\frac{1}{2!}\cdot\left(\frac{f_0}{f}\cdot\frac{1}{4Q_{eff}}\right)^2+\frac{1}{3!}\cdot\left(\frac{f_0}{f}\cdot\frac{1}{4Q_{eff}}\right)^3+\frac{1}{4!}\cdot\left(\frac{f_0}{f}\cdot\frac{1}{4Q_{eff}}\right)^4+\ldots\right] \quad (4)$$

H(jω), when the resonator is operated linearly, is expressed by a first order low-pass filter. For nonlinear operation, it can be modeled as cascaded first-order sections. The number of stages (*N*) can be estimated from the bandwidth ratio of the phase probability distribution for linear and nonlinear regimes. This function can be derived using the slope of the phase transition. On the other hand, *N* can also be obtained directly from the PN results as

$$N=\left(\left|\frac{\partial}{\partial f}(L(f))\right|_{max}\Big/10\right)-1. \quad (3)$$

Alternatively, a truncated exponential-series expansion can be chosen as a simpler closed-form expression for *M(ω)*, where the number of terms is defined by the equivalent number of first-order stages. In this case, the effective quality factor equals Q_{loaded} multiplied by *N*. When the circuit flicker noise is considered, the exponential fitting for *M(ω)* requires the reduction of the flicker corner frequency by the same factor. Under these conditions, the proposed model is defined in (4), with Q_{eff} equal to $N\times Q_{loaded}$ and f_{ceff} as f_c/N.

This model is applied to the measured results of Figure 7. Since the 23MHz (26MHz) oscillator best PN exhibits a maximum slope of -60dB/dec (-40dB/dec), *N* equals five (three), producing an appropriate quantitative description of the results. The average relative error between measurement and model is less than 1%. Figure 8 includes the corresponding Leeson's model prediction using the estimated Q_{loaded} (~ 4,000). As a result, it is observed that the PN performance improves about 15 to 20dB in the 100 – 1000Hz offset-frequency range due to nonlinearity.

CONCLUSIONS

Piezoelectric resonators can be operated in the nonlinear regime when sufficient power is supplied to their electrodes. It has been shown that nonlinearity enhances the PN performance through an appropriate selection of the sustaining amplifier phase-shift.

A new empirical formula based on a truncated exponential-series expansion has been proposed to fit the performance of a nonlinear oscillator. The model includes higher order terms reflecting an effective Q higher than the Q_{loaded}. Additionally, a reduction in the flicker corner frequency has been observed and quantified as the same factor that increases the Q_{loaded}.

The 23MHz IPS mode oscillator presented in this work exhibits -130dBc/Hz at 1KHz offset-frequency operating in air. Such performance improves on previously reported piezoelectric oscillators [10], and is comparable to very-high-Q vacuum-packaged capacitive oscillators at lower frequencies [2, 11] (Table 1).

The proposed PN model reports effective Qs as high as 20,000 producing, to the author's knowledge, the first demonstration of phase noise improvement using the nonlinearity exhibited by a piezoelectric MEMS resonator.

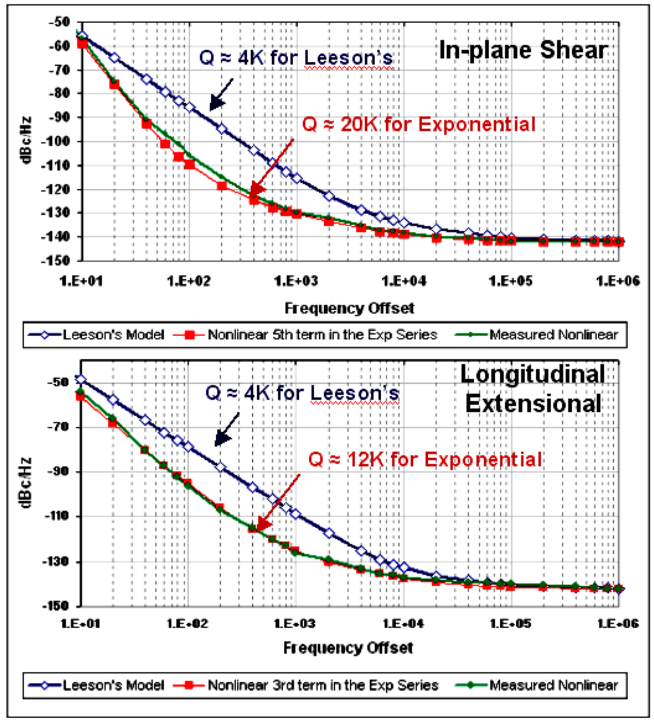

Figure 8: PN Fitting using the exponential series model.

Table 1: Comparison of the 23MHz IPS-mode oscillator with other MEMS-based systems.

Oscillator Frequency	Transduction Mechanism	PN (dBc/Hz) @1KHz*
23MHz (This Work)	**Piezoelectric**	**-96.78**
13.1MHz [2]	Capacitive	-100.29
145MHz [11]	Capacitive	-94.18
1GHz [10]	Piezoelectric	-92.00

*converted phase noise referred to 1GHz carrier frequency

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